Going the Distance

Reporting Category	Measurement	
Topic	Applying formulas to measure attributes of shapes	
Primary SOL	6.10	The student will a) define π (pi) as the ratio of the circumference of a circle to its diameter; b) solve practical problems involving circumference and area of a circle, given the diameter or radius.
Related SOL	6.9	make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system

Materials

- Different Sized Circles handout (attached)
- Scissors
- Yarn
- Inch rulers
- Circling Measures graphic organizer (attached)
- Calculators
- Unit squares

Vocabulary

perimeter of rectangular figure, area of rectangular figure, volume of rectangular figure, circumference of a circle, diameter, radius (earlier grades)

pi, surface area (6.10)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

Note: SOL 5.9 focuses on students being able to describe the circumference of a circle as the distance around a circle. In sixth grade, students are expected to connect and apply their understanding of circumference to the formula $C = 2\pi r$.

1. Open the discussion by asking students for the definitions of *circumference, diameter,* and *radius* of a circle. Tell students that they will now investigate how the circumference of a circle compares to the circle's diameter (and radius).
2. Distribute scissors and copies of the Different Sized Circles handout. Have students cut out the circles.
3. Give each student a ruler, a 3-foot length of yarn, and a copy of the Circling Measures graphic organizer. Direct students to use the yarn to measure the distance around each circle, cutting the exact length of yarn needed for each circle. Then, have students use the ruler to measure the length of each piece of yarn. Instruct them to record each measurement in the chart under "Length of Yarn." Emphasize that this is the **circumference** of each circle.

4. Have students fold each circle in half, crease it, unfold it, and draw a line along the crease. Direct students to use their rulers to measure the length of this line across the center of each circle and record each measurement in the chart under "Length of Line." Emphasize that this is the **diameter** of each circle.
5. Have students divide the diameter of each circle in half and record each value under "Length of Line Divided by 2." Emphasize that this is the **radius** of each circle.
6. Have students divide the length of yarn by the length of line for each circle and record each value under $\frac{\text{Length of Yarn}}{\text{Length of Line}}$. Tell students that they have now determined a relationship between the length of yarn (circumference) and the length of line (diameter)—that is, the ratio of the circumference of a circle to its diameter. Ask them what they observe about the circumference divided by the diameter of each circle. They should notice that each ratio is the whole number 3 followed by different numbers in the decimal places. Point out that they have discovered that the circumference of a circle is a little more than 3 times larger than the diameter of the same circle.
7. Display the formula for circumference, $\boldsymbol{C = 2\pi r}$, and explain each aspect of it as follows:
   - C = circumference (length of yarn)
   - π or pi = the ratio of the circumference of a circle to its diameter (ratio of length of yarn to length of line, or length of yarn divided by length of line)
   - $2r$ = radius multiplied by 2, which is the diameter (length of line)
8. Distribute calculators. In the sixth (blank) column on the Circling Measures graphic organizer, have students write $C = 2\pi r$ in the heading box. Then, have them use calculators to find the exact circumference of each circle by substituting the known values into the formula and performing the indicated operations.
9. Next, have students use unit squares to fill in each circle without going beyond the edges. This will enable them to estimate the area of each circle. Considering that a square does not accommodate rounded edges, point out to students that they will have to estimate the amount of some of the squares being used.
10. After students have completed their estimates of the area of each circle, introduce the formula for the area of a circle, $\boldsymbol{A = 2\pi r^2}$. In the seventh (blank) column on the Circling Measures graphic organizer, have students write $A = 2\pi r^2$ in the heading box. Then, have them use calculators to find the exact area of each circle by substituting the known values into the formula and performing the indicated operations.
11. Have students share how their estimated areas, derived by filling the circles with unit squares, compare with the exact areas calculated by using the formula.

Assessment

- **Questions**
  - What is the term for the distance around a circle? What is the term for the distance around a rectangular figure?
  - What is the relationship between the diameter and the radius of a circle?
  - What is the relationship between the circumference of a circle and its diameter or radius?

- **Journal/Writing Prompts**
  - Explain how yarn was used to measure the circumference of circles.
  - Given the proportional relationship between circumference and diameter, explain why $\frac{C}{d} = \pi$ is true.
- **Other**
  - Develop an activity similar to this one to apply to perimeter and area of triangles and rectangles.

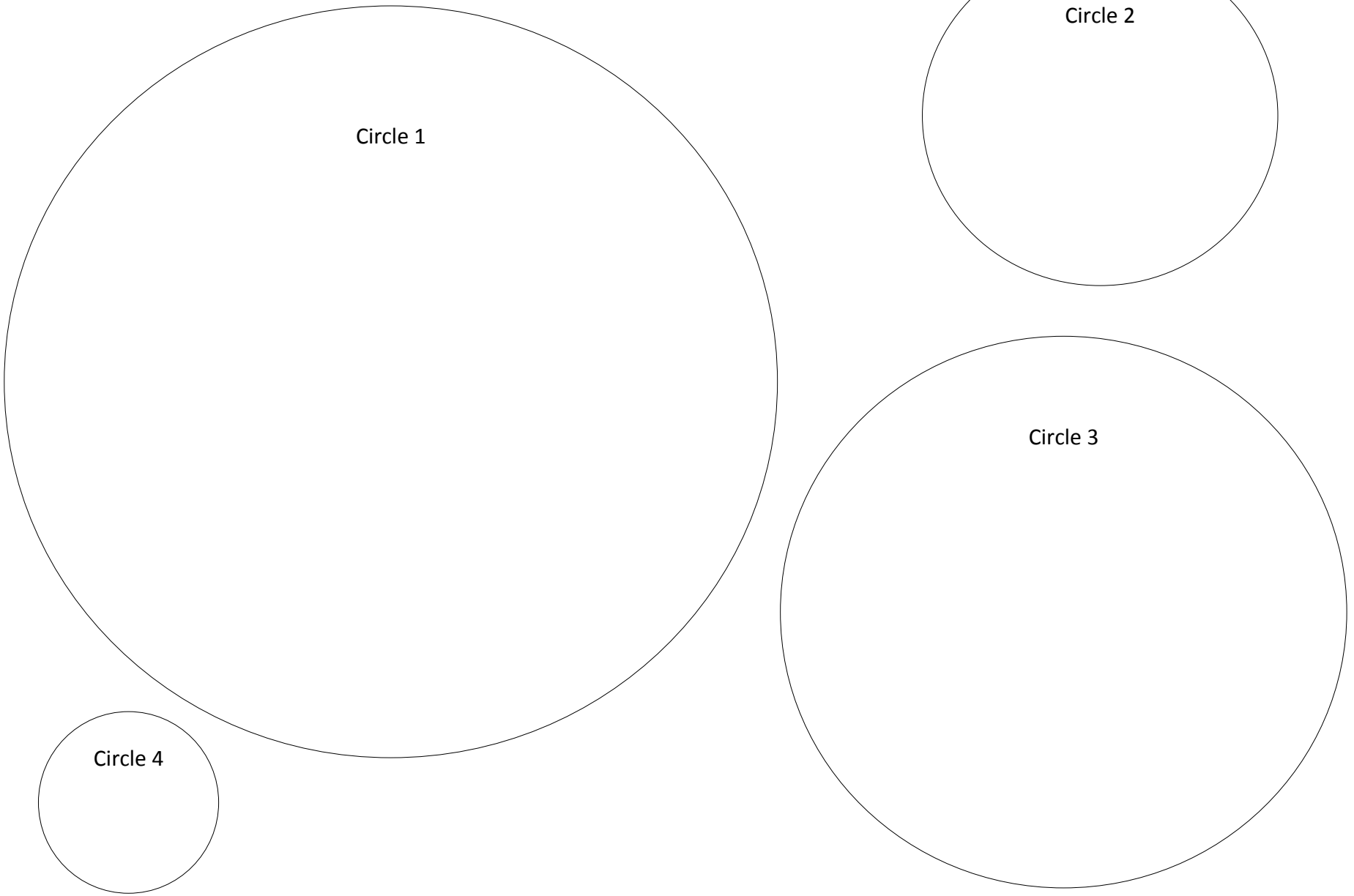
Extensions and Connections (for all students)

- Have students do this activity with other shapes.

Strategies for Differentiation

- Provide students with pieces of yarn already cut to the length of the circumference of each circle.
- Demonstrate the process of using yarn to measure the circumference of one of the circles.
- Demonstrate the calculations for one of the circles.
- Clue students to recognize the relationships between the various values in the chart.

Different Sized Circles



Circling Measures

Name _____ **Date** _____

Circle	Length of Yarn (Circumference)	Length of Line (Diameter)	Length of Line Divided by 2 (Radius)	$\frac{\text{Length of Yarn}}{\text{Length of Line}}$ (pi)		
1						
2						
3						
4						